ENTRANSY

- A CORE QUANTITY IN HEAT TRANSFER

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Outline

1. **Why** should we introduce the new quantity, *entransy*?

2. **How** to introduce the entransy?

3. **Where** can the entransy be applied?

4. **What are differences** between entransy & entropy?
1. Why should we introduce entransy?

Existing method of heat transfer optimization

Prigogine (1947) developed the minimum entropy generation (MEG) principle for transport processes. Bejan (1979) extended this principle to the optimization of heat transfer, “called thermodynamic optimization”.

However, we find that MEG method is not always applicable to the optimization of heat transfer process.
Optimization of Heat Conduction (V-P problem)

- Heat delivered from heat sources inside the domain is removed via conduction through two points at $T_1$ and $T_2$ to the environment.

- Find the optimal distribution of the thermal conductivity, $k(x, y)$ with fixed overall thermal conductivity to make the domain the lowest average temperature.
Optimization of Heat Conduction (V-P problem)

For $k(x,y)$ based on the MED principle, the domain temperature is reduced at first with $T_2$ going down.

However, the domain temperature does not decrease, but it increase with $T_2$ going down, when $T_2 < 240$K.
2. How to introduce entransy in HT?

Analogy method

Since the 19th century the analogy between heat and electric conductions, as a mature approach, has already been widely applied, including solving complex heat conduction problems.

However, a physical quantity is still missing in the analogy between various corresponding quantities in heat and electric conductions.
### 2.1 Analogy method between heat and electric conduction

<table>
<thead>
<tr>
<th></th>
<th>Transport law</th>
<th>Transport equation</th>
<th>Transferred non-dissipative quantity</th>
<th>Potential</th>
<th>Stored dissipative quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Electric conduction</strong></td>
<td>Ohm’s law</td>
<td>1-D eq.</td>
<td>Electrical charge</td>
<td>Electric voltage</td>
<td>Electric poten. energy</td>
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<tr>
<td></td>
<td>( i = -\sigma \frac{\partial V}{\partial x} )</td>
<td>( \dot{Q}_{ve} = \frac{\Delta V}{R_e} )</td>
<td>( Q_e )</td>
<td>( V )</td>
<td>( Q_{ve} \frac{V}{2} )</td>
</tr>
<tr>
<td><strong>Heat conduction</strong></td>
<td>Fourier’s law</td>
<td>1-D eq.</td>
<td>Heat</td>
<td>Temperature</td>
<td></td>
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<tr>
<td></td>
<td>( \dot{q} = -\lambda \frac{\partial T}{\partial y} )</td>
<td>( \dot{Q}_{vh} = \frac{\Delta T}{R_h} )</td>
<td>( Q_h )</td>
<td>( T )</td>
<td>( Q_{vh} \frac{T}{2} = G ) (for ( C = \text{const} ))</td>
</tr>
</tbody>
</table>

- **Lack a quantity corresponding to the electric potential energy:** \( Q_{vh} \frac{T}{2} = G \)
- -- entransy of a solid object
2.2 physical meaning of entransy (1)

According to Einstein’s mass-energy relation

- **Thermomass:** \( M_h = \frac{Q_{vh}}{c^2} [kg] \)
  
  \( Q_{vh} \) Thermal energy
  
  \( c \) Light speed
  
  \( C \) Heat capacity

- **Potential energy of thermomass** in a temperature field:

  \[ E_h = \frac{Q_{vh} T}{2} \frac{C}{c^2} [J] \]

  Removing \( C/c^2 \) leads to

  \[ G = \frac{Q_{vh} T}{2} [J \times K] \]

- **Entransy is nothing but the simplified expression of the thermomass energy.**

"Guo, et.al., Int J Heat Transfer, 2007"
2.3 Physical **meaning** of Entransy (2)

It is for the **energy nature** of entransy that

- **Entransy**, \( G = Q_{vh} T/2 \), of a solid object represents its **ability of heat transfer during a time period**, as the electric potential energy of a battery represents its **charge transfer ability**.
2.4 Deductive method

- Energy conservation in solids: 
  \[ MC \frac{dT}{dt} = \nabla \cdot Q \]

- Multiplying both sides by \( T \) leads to the balance equation of entransy

  Entransy variation \quad Entransy transfer \quad Entransy dissipation rate

\[
\frac{d}{dt} \left( \frac{1}{2} Q_{vh} T \right) = \nabla \cdot (QT) + Q \cdot \nabla T
\]

- Heat (mass nature) is conserved, while entransy (energy nature) is dissipated during transport process, which can measure the heat transfer irreversibility too.

\[ Q_{vh} = MCT \]
2.5 Principle of minimum ED thermal resistance

- The thermal resistance $R_h = T / \dot{Q}$ is valid for 1-D problem only, because of arbitrariness of definition for non-uniform temperature boundary.

- It is possible to define the thermal resistance based on the entransy dissipation for 3-D problems

  $$R_d = \frac{\dot{\Phi}_g}{\dot{Q}^2}, \quad \dot{\Phi}_g \quad -- \text{entransy dissipation rate}$$

- The principle of MED based thermal resistance states: “the performance of Heat transfer without heat-work conversion is optimized, if the MED based thermal resistance is minimized”.
3. Where can the entransy be applied?

3.1 Heat conduction optimization

- Find optimal $k(x,y)$ to minimize the domain temperature

- The average temperature decreases linearly with decreasing $T_2$ based on Minimum EDTR.
3.2 Heat convection optimization

Based on the Minimum EDTR Principle, we can obtain the optimal velocity field with multi-vortexes for best heat transfer performance at fixed pumping power.

Mass production of CO₂ Water Heater in the heat pump by Daikin Company, Japan.

Weight: from 6.5 kg down to 6.0 kg
Material saving: 8%

Meng, et.al., Int J Heat Transfer, 2005
For increasing the energy efficiency, the principle of minimum entransy-dissipation-based thermal resistance has been also applied in the optimization of thermal system, including:

Heat exchanger couple;
Heat exchanger network;
Thermal radiation system.

Chen, et.al., Int J Heat Transfer, 2009
4. What are the **differences** between entransy & entropy?

<table>
<thead>
<tr>
<th>State quantity</th>
<th>Entropy</th>
<th>Entransy</th>
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<tbody>
<tr>
<td><strong>State quantity</strong></td>
<td>$S=S(T,V)$</td>
<td>$G=G(T)$</td>
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<tr>
<td></td>
<td>$dS=Mc_vdT/T+RdV/V$</td>
<td>$dG=McTdT$</td>
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<td>$G=McT^2/2$ (for c=const)</td>
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<tr>
<th>Physical meaning</th>
<th>Ability of heat to work conversion</th>
<th>Ability of heat transfer rate</th>
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<tr>
<th>Process quantity</th>
<th>Entropy flow</th>
<th>Entransy flow</th>
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<td><strong>Process quantity</strong></td>
<td>$Q/T$</td>
<td>$QT$</td>
</tr>
<tr>
<td></td>
<td>Entropy generation rate for thermodyn. process</td>
<td>Entransy dissipation rate for heat transfer process</td>
</tr>
<tr>
<td></td>
<td>$\dot{s}_g = \frac{\kappa}{T^2} \left( \frac{dT}{dx} \right)^2$</td>
<td>$\dot{g}_d = \kappa \left( \frac{dT}{dx} \right)^2$</td>
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<th>Process irrev. and optim. Criterion</th>
<th>Least action principle</th>
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<td>Minimum entransy dissipation rate leads to Fourier’s conduction law</td>
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ENTRANSY IS A CORE PHYSICAL QUANTITY IN HEAT TRANSFER

Because the entransy -- the ability of heat transfer and the entransy dissipation rate
  a) is the action of heat conduction;
  b) can measure the heat transfer irreversibility;
  c) is the optimization criterion of heat transfer.
Thank you very much for your attention!
Finlayson (1972) pointed out that Fourier’s law can not be derived from the principle of minimum entropy generation unless the thermal conductivity $k \sim 1/T^2$, which is inconsistent with the actual situation!
2.2 physical meaning of entransy (1)

Based on thermomass inertia a general conduction law has been derived, which can be reduced to Fourier’ conduction law and various non-Fourier’s equations under different simplifications.

Guo, et.al., Int J Heat Transfer, 2007
3.4 Least action principle in heat transfer

Fourier’s law with constant thermal conductivity can be derived based on the principle of minimum entransy dissipation, rather than the principle of minimum entropy generation.

The principle of MED is the least action principle in heat transfer, which states that Fourier’s law can be derived as the entransy dissipation rate in the domain reaches its minimum.

This is why the entransy dissipation rather than the entropy generation is the optimization criterion of heat transfer without heat-work conversion.
For a counter-flow heat HX its effectiveness increases with the increase of entropy generation as it is smaller than 0.5.

R. K. Shah et al. analyzed the irreversibility of 18 heat exchanger flow arrangements and concluded “the concept of minimum entropy generation associated with the maximum efficiency was not quite applicable to heat exchanger analyses”.

Shah, et.al., ASME J Heat Transfer, 2004
3.3 Heat exchanger optimization

Effectiveness versus EG

- Unlike the entropy analysis for cross-flow HX, its effectiveness increases monotonously with the decrease of EDTR, i.e., the entransy approach is more reasonable than the entropy approach.

Guo, et.al., Int J Heat Transfer, 2010
3.2 Heat convection optimization

- Find optimal $u(x,y)$ to minimize the domain temperature.
- The average domain temperature based on Minimum EDTR is lower than that based on Minimum entropy generation.

Heat convection optimization with stirring

- $T_1 = 300$ K
- $T_2 = 550$ K
- $Q = 70$ W
- $T_{m1} = 580$ K
- $T_{m2} = 614$ K

Chen, et.al., Int J Heat Transfer, 2013
3.3 Optimization of thermal systems

- For increasing the energy efficiency, the principle of minimum entransy-dissipation-based thermal resistance (EDTR) has been applied in the optimization of thermal systems, including:

  - District Heating System (Total Heat transfer area decreases 16%)
  - Absorption Energy Storage System (heat storage rate increases 13%)
  - Integrated Thermal & Power System (Wind curtailment reduces 27%)

References:
Least action principle in heat transfer

- Not only can the entropy generation or exergy destruction measure heat transfer irreversibility, but entransy dissipation or other quantities being the function of temperature can also measure it.

- The entransy dissipation is the unique action of heat transfer. Its extreme leads to Fourier’s conduction law. This is why the entransy dissipation can be taken as the optimization criterion of heat transfer.

Exergy is a non-conserved quantity during a heat transfer process. Its destruction measures irreversibility of heat transfer process being part of a power cycle, and corresponds to the efficiency of heat-work conversion.

Entransy is also a non-conserved quantity during a heat transfer process. Its dissipation measures irreversibility of heat transfer not being part of a power cycle, and corresponds to the efficiency of object heating or cooling.
On the necessity of $G(2)$

- The gap in the existing heat transfer analyses is that there is no optimization criterion of heat transfer not involving in a thermodynamic cycle.

- The principle of MED thermal resistance is the least action principle in heat transfer, by which Fourier’s law can be derived. This is why entransy dissipation can be taken as the optimization criterion of heat transfer.

**On the physical meaning of entransy**

- **Entransy** of an object represents its heat transfer ability during a time period, as the electric energy of a battery represents its charging ability.

- Therefore, the entransy dissipation is essentially the thermomass energy dissipation during heat conduction.

- According to the law of energy conservation, the thermomass energy will dissipate into an another form of energy with lower level, which might be the “dark energy” in the astrophysics.
On the entropy generation

- Entropy generation in 1-D steady heat conduction

\[ \frac{Q}{T_h} - \frac{Q}{T_c} = S_g \]

- No one knows where the entropy generation comes during the irreversible heat conduction.

- That means, the entropy generation comes from thin air during the irreversible heat conduction.

- This is why we have only the balance equation, rather than the conservation equation for entropy.
If according to Bejan’s logic, “the ‘dissipated entransy’ is simply a multiple of the generated entropy”, the generated entropy is nothing but a multiple of temperature, so entropy is not new physical quantity either.

All other quantities in thermodynamics, except temperature, pressure, specific volume, are not new or useful because they can all be expressed by these three quantities as well!
“Heat-electric analogy” is a common and widely used method for heat transfer analysis.

However, the analogy between the heat and electric conduction was regarded as “heat-work analogy” and argued that it is perpetual motion machine and violates the second law of thermodynamics.

Since there is no heat-work conversion in the heat and electric conduction processes, such an analogy is not the “heat-work analogy” at all, and not to mention that this analogy violates the second law of thermodynamics.
The field synergy number $Fc$ and the Stanton number $St$ have identical formulas. However, their physical meaning and applications are quite different.

$St$ comes from the analogy between heat and momentum transfer. The group $StPr^{2/3} = C_f/2$ was called Reynolds-Colburn analogy. It can be used directly to infer heat transfer data from measurements of the shear stress.
On the field synergy number（2）

- $F_c$ comes from the integration of dot production of the velocity vector and the temperature gradient vector over the whole domain.

- $F_c$ stands for the dimensionless heat source strength over the entire domain and physically is the indication of the synergy degree.

- The larger the $F_c$, the better the synergy between the velocity and temperature gradient fields, the better the heat transfer performance.
On the physical meaning of entransy

- Entransy \( G = MC_v T^2 / 2 \) is a simplified expression of the thermomass energy, \( E_h = MC_v^2 T^2 / 2c^2 [\text{J}] \), where \( E_h \) has the dimension of energy.

- Therefore, the entransy dissipation is essentially the thermomass energy dissipation during heat conduction.

- According to the law of energy conservation, the thermomass energy will dissipate into an another form of energy (might be the dark energy) with lower level.
On the duplicate of entropy

- If according to Bejan’s logic, “the ‘dissipated entransy’ is simply a multiple of the generated entropy”, the generated entropy is nothing but a multiple of temperature, so entropy is not new physical quantity either.

- All other quantities in thermodynamics, except temperature, pressure, specific volume, are not new or useful because they can all be expressed by these three quantities as well!
On the violation of 2\textsuperscript{nd} law

- The analogy between the heat and electric conductions was regarded as “heat-work analogy” and argued that it is false and violates the second law of thermodynamics.

- Since there is no heat-work conversion in the heat and electric conduction processes, such an analogy is not the “heat-work analogy” at all, and not to mention that this analogy violates the second law of thermodynamics.
For an electric capacitor with the electric charge $Q_{ve}$ and the electric capacity of $C$ at the electric voltage $V$ its potential energy is

$$dE = Q_{ve}dV,$$

because of $Q_{ve} = CV$

$$E = CV^2/2$$

For a constant volume object with the thermal energy of $Q_{vh}$ and the thermal capacity of $C$ at the temperature $T$ its potential energy is

$$dG = Q_{vh}dT,$$

because of $Q_{vh} = CT$

$$G = MCT^2/2$$
On the $T - Q$ Diagram

- The entransy transfer rate out of the hot stream during heat transfer is equal to the area under temperature line 1–2.

- The total entransy transfer rate into the cold stream is represented by the area under temperature line 3–4.

- Therefore, the difference between these two areas, as represented by the shaded area, is the net entransy dissipation rate, $\Phi_g$, during heat transfer.
2.3 Why called $G_h$ entransy?

- Clausius coined the word **en-tropy** for $S = \left( \frac{Q}{T} \right)$ because it possesses both the nature of **energy** and **transformation** ability.
  
  *en* - prefix of energy;  
  *tropy* [τροπη] - root of transformation

- We have coined the word **en-transy** for $G = \frac{Q_{eh} T}{2}$ because it possesses both the nature of **energy** and **transfer** ability.
  
  *en* --- prefix of energy;  
  *transy* -- root of transport

*Guo, et.al., Int J Heat Transfer, 2007*
3.4 Performance analysis method for a Heat Exchanger

- Analytical relation between the effectiveness and EDTR

\[
\varepsilon = \frac{2}{2R^* + (1 + C)} \quad \text{or} \quad R^* = \frac{1}{2} \left( \frac{1}{C + 1} \right) \quad C^* = \frac{C_{\min}}{C_{\max}}
\]

- Unlike the LMTD method or the \( \varepsilon \)-NTU method, the above relation (\( \varepsilon \) - EDTR method) holds for heat exchangers with different flow arrangement.

- The EDTR is an unified performance evaluation criterion for the heat exchanger with different flow arrangement from the viewpoint of heat transfer irreversibility.

Guo, et.al., Int J Heat Transfer, 2010